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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1223

SOME EXPERIENCES REGARDING THE NONLINEARITY OF HOT WIRES

By R. Betchov and W. Welling

Translation of "Quelques expériences sur la non-linéarité des fils chauds." Koninklijke Nederlandse Akademie van Wetenschappen. Mededeling No. 66 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hogeschool te Delft. Reprinted from Proceedings Vol. LIII, No. 4, 1950.

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We compare here the results of some experiences with the formulas established in our preceding report "Nonlinear Theory of a Hot-Wire Anemometer." We shall show that the nonlinear term plays a role as important as the thermal conduction in the calculation of the thermal inertia of the hot wire.

I. INTRODUCTION

According to our nonlinear theory[1] the equation of the hot wire must contain the terms expressing:

(a) The heat transfer from the wire to the air in proportion to the temperature T

(b) The heat transfer in proportion to T^2 (nonlinearity)

(c) The heat conduction at the ends of the wire

(d) The thermal inertia due to the specific heat and the mass of the metal

We shall study first the effects (a), (b), and (c) in treating the case of 11 wires of small diameter (2 microns), and then turn to the effect of inertia. We refer without further specifications to the formulas of the nonlinear theory, numbered from 1 to 73, and shall continue with the number 74.

II. PREPARATION OF THE WIRES

We prepared our wires by utilizing Wollaston wire of platinum, bent in U form and soldered to a support before being cleaned. In

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[1]Betchov, R.: Théorie non-linéaire de l'anémomètre à fil chaud, (Meded. 61). Proc. Kon. Ned. Adad. v. Wetensch. Amsterdam, 52, 1949, pp. 195-207.

order to remove the silver, we used a jet of an acid solution (50-percent distilled water, 50-percent HNO_3) and electrolysis with a current of 5 to 20 milliamperes. Figure 1 indicates the arrangement used.

The jet does not break the wire. Nevertheless, if the diameter is smaller than 5 microns, the dust particles entrained by the liquid are dangerous and the solution must be filtered each time before using. The flask is mounted on a support which can be precision adjusted. Figure 1 shows the sequence of operations viewed under the microscope.

One can obtain extremely short wires by displacing the jet perpendicularly to the wire, so as to remove the silver, sometimes in front, sometimes at the rear. The cleaned wire is rinsed with ordinary water and brought to a faint incandescence so as to permit microscopic examination. Only wires that redden in a regular and symmetrical manner are used.

III. STATIC CALIBRATION

We give here the results obtained with 11 wires of platinum with lengths between 0.25 and 1.6 millimeters and with diameters of about 2 microns. Figure 2 indicates the cold resistances and the lengths and shows the order of magnitude of the individual variations.

Every wire had been calibrated with air streams of 2 to 10 meters per second and we studied the magnitude

$$H = \frac{RI^2}{R - R_o} \tag{74}$$

as a function of the ratio R/R_o. Extrapolating starting from the measured values of H, we determined the limiting value H_o, corresponding to the case $R/R_o = 1$.

Our wires gave 11 values of H_o for $V = 2$ meters per second which we indicated as functions of the wire lengths in figure 3. The theory yields for I tending toward zero

$$H_o = \frac{A}{1 - \tanh \xi_o / \xi_o} \tag{75}$$

with

$$\xi_o = \frac{l}{l_o^*}, \quad l_o^* = \sqrt{\frac{\chi\sigma}{\alpha S_o A}} \tag{76}$$

and we plotted the theoretical curves corresponding to the values $l_o^* = 6 \times 10^{-2}$ millimeter and 8×10^{-2} millimeter. One can see that, if l tends toward infinity, H_o tends toward the value 500 milliamperes^2. King's formula gives us, for $d = 2.1$ microns, $A = 500$ milliamperes^2 and we thus can see that the effect of conduction at low temperature corresponds to the theory.

When the resistance increases, the effect of thermal conduction tends to make H diminish. We take as example the wire No. 8, of a length of 0.75 millimeter. Figure 4 gives us the experimental values of H, measured twice, with a one-day interval. Taking into account only the conduction, one obtains for H the dotted curve, according to

$$H = A \frac{1 - (I^2/A) \tanh \xi/\xi}{1 - \tanh \xi/\xi} \tag{77}$$

The solidly drawn curve was calculated taking into consideration the nonlinearity according to formula (36) with the coefficients $\gamma = 1.14 \times 10^{-3}$, $A = 450$ milliamperes^2, $l_o^* = 7 \times 10^{-2}$ millimeters. We see that it corresponds to the experience of the first day, and that one has about $H_o = 555$. The values of H measured the next day are lower.

Probably the differences are caused by dust particles which have settled on the wire during that time interval and produce an enlargement of the region of immobile air around the wire, thus reducing the transport of heat by the air stream.

We studied the increase of H with the temperature and for every wire treated we measured the ratio

$$h = \frac{H\left(\frac{R}{R_o} = 2\right) - H_o}{H_o} \tag{78}$$

This ratio can be calculated and figure 5 shows the experimental and theoretical results.

The theory seems satisfactory to us, in spite of the deviations of the points.

IV. DYNAMIC CALIBRATION

We measured the response of a hot wire to fluctuation of the electric current. For that purpose, the wire was placed in a bridge (fig. 6) fed by the plate flow of a pentode. The heating current can be modulated with the aid of a low-frequency oscillator; the alternating intensity i is indicated by a special apparatus. We had $R_1 = 100$ and $R_2 = 1,000$ ohms; the self-induction L compensated both the self-induction of the line leading to the hot wire and its ohmic resistance. The bridge electromotive force was applied to an analyzer which transmitted only the signal of the frequency of the oscillator, permitting operation without impediment by turbulence. The filtered signal was transmitted to a cathodic oscillograph which enabled us to balance the bridge for the frequency used.

Since analyzer and oscillograph were grounded, it was necessary to especially insulate the feeding system.

Actually, the rectifier and the oscillator represent, normally, with respect to the alternating network, a capacity of about 5,000 $\mu\mu f$; this network is always grounded at some point which introduces an undesirable element into the circuit. We eliminated this inconvenience by using a transformer which has a weak capacity between the primary and the secondary.

In order to eliminate the skin effect, we had to employ a special line leading to the hot wire. In this manner, the bridge proved satisfactory from 0 to 75 kilocycles. The impedance of the circuit $R'C'$ has the purpose of compensating the fluctuations of resistance of the wire and the calculation shows that when the bridge is balanced the electromotive force rI is proportional to the electromotive force at the boundaries of R'. The measurements were made in the following manner:

1. The wire is placed in the tunnel and subjected to the air stream, with $i = 0$ and $R' = 0$. One then adjusts R_3 so as to balance the bridge. A galvanometer (not represented in the figure) is used for that purpose.

2. In modulating the current, with i of the order of 3 percent of I, and at a low frequency, one adjusts R' and C' in such a manner as to balance the bridge for alternating current. The values of R' and C' as well as the frequency are noted.

3. The same procedure is followed with increasing frequencies f, up to about 10,000 periods.

The product $2\pi fR'C'$ gives the tangent of the angle of phase displacement between the alternating current traversing the wire and the variation of its resistance. This phase displacement amounts to 45° for a certain value $f_{45^{\circ}}$ of the frequency which we compare (a) with the theoretical value f^{*} for the linear and infinite wires (51), (b) with the value f^{**} of our nonlinear theory, and (c) with the value given by Dryden

$$f_{\text{Dryden}} = \frac{1}{2\pi} \frac{\alpha S_o}{mc} \frac{R_o I^2}{R - R_o} \tag{79}$$

deduced from equation (51) by replacing A by $RI^2/(R - R_o)$. We have here the general relation $\omega = 2\pi f$.

In order to calculate these theoretical values, one must know the constant

$$\frac{\alpha S_o}{mc} = \frac{16\alpha\rho}{\pi^2 d^4 c\delta} \tag{80}$$

It can be seen that an error of 5 percent concerning the diameter of the wire results in an error of 20 percent concerning that constant; thus it is preferable to determine it empirically. The calculation of that constant gave us, in the case of the wire No. 8, values that were too low; we multiplied it arbitrarily by a factor 1.26, so as to make theory and experience coincide if the temperature of the wire is low. Figure 7 indicates the measured quantities $f_{45^{\circ}}$ as functions of the ratio R/R_o, as well as the calculated curves.

One can see that the empirical results are intermediate between f^{**} and f_{Dryden}. In order to explain this, one must take into account the variation with the temperature, the specific heat of the metal, the product $\alpha\rho$, and the density. With the aid of the International Critical Tables, we estimated that the constant $\alpha\rho/mc$ diminishes according to the approximate formula

$$\left(\frac{\alpha\rho}{mc}\right)_R = \left(\frac{\alpha\rho}{mc}\right)_{R_o} \left\{1 - \epsilon\left(\frac{R}{R_o} - 1\right)\right\} \tag{81}$$

with $\epsilon = 0.06$ in the case of platinum. This correction reduces the calculated frequencies. We showed on figure 7 the curve obtained from f^{**} and indicated "f^{**} corrected." This correction should be applied equally to f_{Dryden} and could still increase the differences stated before. Other wires give analogous results, and we estimate that our nonlinear theory corresponds to experience.

After having studied the frequency giving a phase displacement of 45° we must examine the behavior of the wire when the frequency is lower or higher. The tangent of the phase-displacement angle φ equal to the ratio of the imaginary and the real part of the electromotive force rI is given by the product $2\pi R'C'f$, and the product $R'C'$ should be independent of the frequency - in absence of thermal conduction - if the formula (67) were exact.

When f is smaller than f_{45}, the amplitude corresponds to the calculated values (formula 66) which is normal since this result depends only on the derivative of R with respect to I, and the theory gives the correct values of H.

The product $R'C'$ is constant - as shown in figure 8 - as long as f is smaller than f_{45}, but the calculated values are slightly lower than the measured ones.

Beyond f_{45}, the amplitude diminishes according to the theory and the formula (64) is verified, but the product $R'C'$ decreases more rapidly than was foreseen in the theory.

The dotted curves of figure 8 were calculated from the formula (62) and it can be demonstrated that, for frequencies tending toward infinity, the phase displacement tends toward

$$\tan\varphi = \frac{1 - \frac{BI^2}{A - I^2 + BI^2}\frac{1}{\sqrt{E\xi}}}{1 - \frac{BI^2}{A - I^2 + BI^2}}\,\xi\sqrt{2}\sqrt{\frac{f}{f*}} \tag{82}$$

It seems, therefore, that this abnormal phase displacement is even more pronounced than was expected according to experience; however, the experimental errors may considerably affect our results, particularly the error concerning R_3. Also, it must be noted that, when the tangent varies, for instance from 10 to 20, the angle varies only from 84° to 87° which reduces the importance of this effect.

Finally, we have carried out a few preliminary tests with tungsten wires and have found the quantity H to be remarkably constant, regardless of the length of the wire.

Translated by Mary L. Mahler
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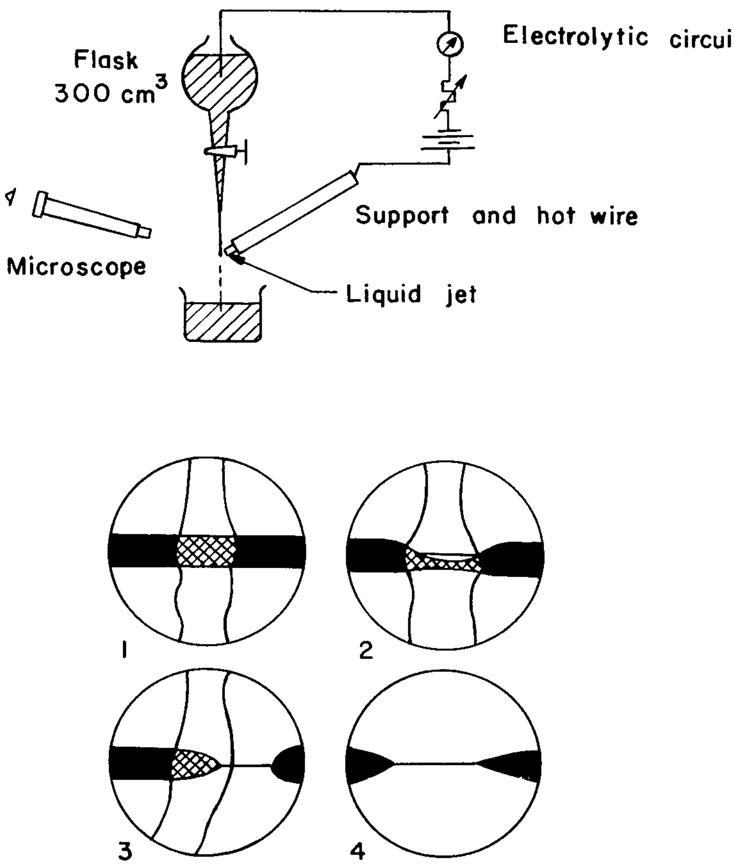

(1) Start (2) The platinum appears
(3) The jet forms a point (4) Finished wire

Figure 1.- Preparation of the Wollaston wires.

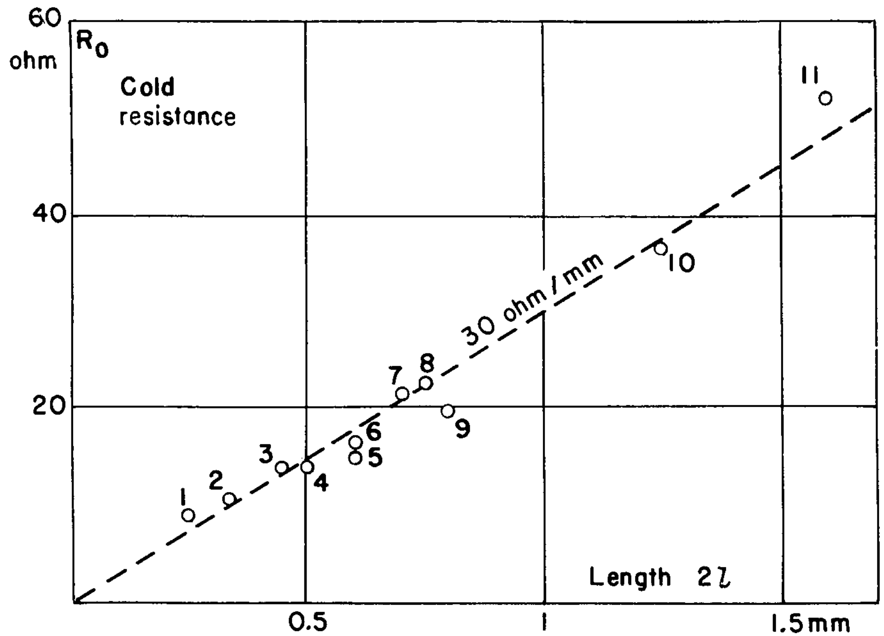

Figure 2.- Cold resistances for lengths of the wires.

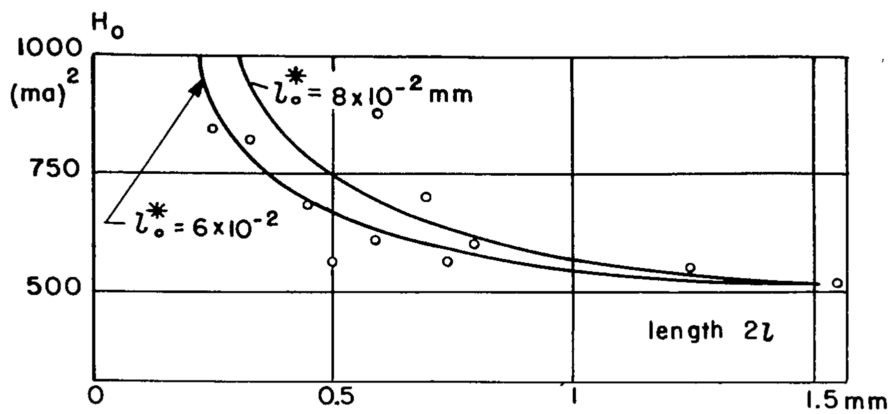

Figure 3.- Effect of conduction at low temperature.

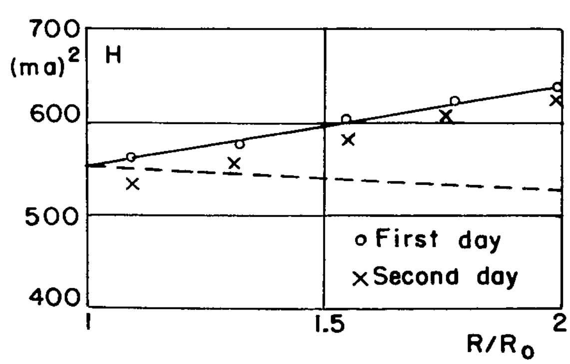

Figure 4.- Effect of nonlinearity.

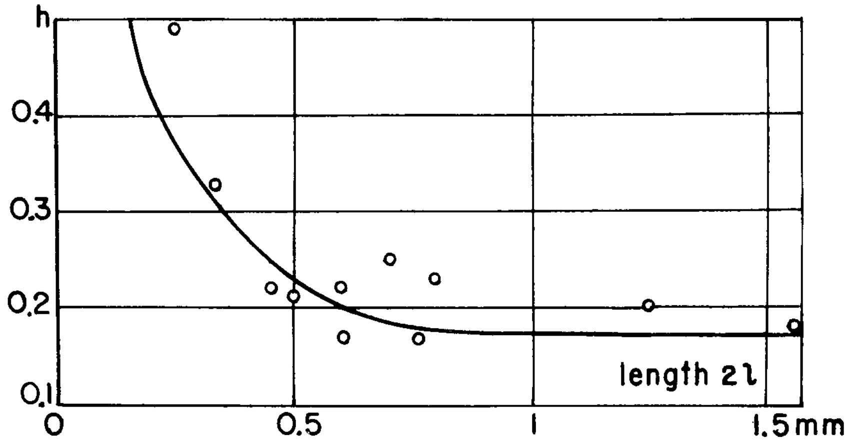

Figure 5.- Effect of the length on the ratio h.

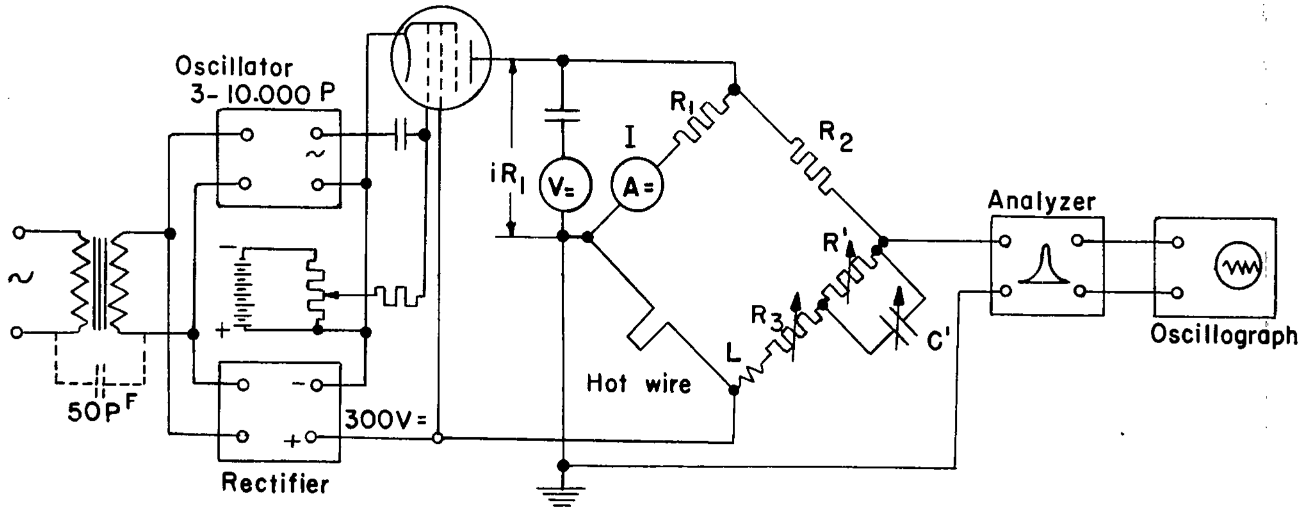

Figure 6.- Measurement of the thermal inertia of a hot wire.

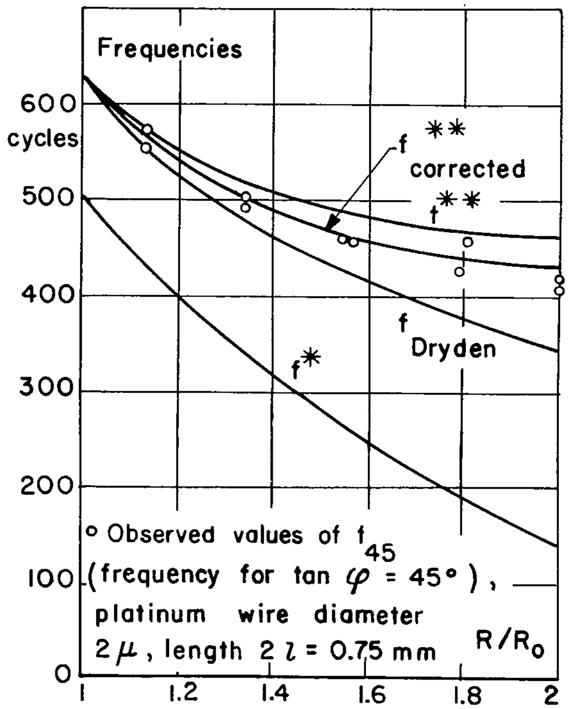
Frequencies
600
cycles
500
400
300
200
100
0
1
1.2
1.4
1.6
1.8
2
f** corrected
f**
f Dryden
f*
o Observed values of f_{45} (frequency for tan φ = 45°), platinum wire diameter 2 μ, length 2 l = 0.75 mm
R/R$_0$

Figure 7.

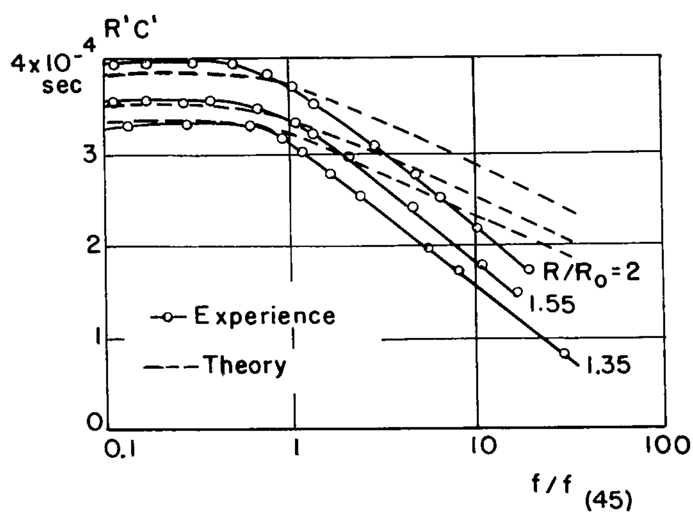

Figure 8.- Anomalies of the phase displacement.